Test Code : RC (Short Answer Type) 2006

JRF in Computer and Communication Sciences

The Candidates for Junior Research Fellowship in Computer Science and Communication Sciences will have to take two tests - Test MIII (objective type) in the forenoon session and Test RC (short answer type) in the afternoon session. The RC test booklet will have two groups as follows:

GROUP A

A test for all candidates in logical reasoning and basics of programming, carrying 20 marks.

GROUP B

A test, divided into five sections, carrying equal marks of 80 in the following areas:

(i) Mathematics, (ii) Statistics, (iii) Physics at M.Sc. level, (iv) Radiophysics/ Telecommunication/ Electronics/ Electrical Engg., and (v) Computer Science at M.Sc./ M.E./ M.Tech. level.

A candidates has to answer questions from *only* one of these sections, according to his/her choice.

The syllabi and sample questions of the RC test are given overleaf.

Syllabus

Elements of Computing:

Logical reasoning, basics of programming (using pseudo-codes), Elementary data types and arrays.

Mathematics:

Graph theory and combinatorics: Graphs and digraphs, paths and cycles, trees, Eulerian graphs, Hamiltonian graphs, chromatic numbers, planar graph, tournaments, inclusion-exclusion principle, pigeon-hole principle.

Linear programming: Linear programming, simplex method, duality.

Linear algebra: Vector spaces, basis and dimension, linear transformations, matrices, rank, inverse, determinant, systems of linear equations, characteristic roots (eigen values) and characteristic vectors (eigen vectors), orthogonality and quadratic forms.

Abstract algebra: Groups, subgroups, cosets, Lagrange's theorem, normal subgroups and quotient groups, permutation groups, rings, subrings, ideals, integral domains, fields, characteristic of a field, polynomial rings, unique factorization domains, field extensions, finite fields.

Elementary number theory: Elementary number theory, divisibility, congruences, primality.

Calculus and real analysis: Real numbers, basic properties, convergence of sequences and series, limits, continuity, uniform continuity of functions, differentiability of functions of one or more variables and applications, indefinite integral, fundamental theorem of calculus, Riemann integration, improper integrals, double and multiple integrals and applications, sequences and series of functions, uniform convergence.

Integral transforms: Laplace transform, Fourier transform.

Differential equations: Solutions of ordinary and partial differential equations and applications.

Statistics:

Probability Theory and Distributions: Basic probability theory, discrete and continuous distributions, moments, characteristic functions, Markov chains.

Estimation and Inference: Sufficient statistics, unbiased estimation, maximum likelihood estimation, consistency of estimates, most powerful and uniformly most powerful tests, unbiased tests and uniformly most powerful unbiased tests, confidence sets.

Linear Models: Gauss-Markov set up and least squares theory, multiple linear regression, one and two way analysis of variance.

Multivariate Analysis: Multivariate normal distribution, Wishart distribution, Hotelling's T^2 test, principal component analysis, multiple and canonical correlations, discriminant analysis, cluster analysis, factor analysis.

Physics:

Classical Mechanics: Variational principle and Lagrange's equation, Central force problem, Rigid body motion, Hamilton Equation of motion, Canonical transformations, Hamilton Jacobi theory and Action Angle variables, Lagrangian and Hamiltonian formulation for continuous Systems and Fields, relativistic mechanics.

Electrodynamics: Electromagnetic fields and Potentials, electromagnetic radiation, scattering, dispersion, relativistic electrodynamics.

Thermodynamics and Statistical Mechanics: Reviews of thermodynamics, statistical basis of thermodynamics, density matrix formulation, ensembles, partition function and its uses, Maxwell-Boltzmann, Bose-Einstein and Fermi Dirac statistics, simple gases, Ising Model.

Non-Relativistic Quantum Mechanics: Basics of quantum mechanics, the two body problem and central potential, quantum particles in electromagnetic fields, matrix mechanics and spin, approximative methods: stationary states, approximative methods: time dependent problems.

Solid State Physics: Crystal structures, interacting forces, lattice vibrations, electronic band structures, density of states, elementary excitations, transport properties.

Electronics: Basics of semiconductor physics, amplifiers, communication principles.

Vibrations and Waves: Forced vibrations, coupled vibrations, stretched strings, small oscillations.

Atmospheric Physics: Fundamental laws of forces/ motion and their conservation meant for atmospheric physics, thermodynamic energy equations, circulation and scale analysis of vorticity, planetary boundary layer, atmospheric oscillation and perturbation theory, numerical analysis and prediction of weather and climatology.

Radiophysics/Telecommunication/Electronics/Electrical Engg.:

Boolean algebra, digital circuits and systems, circuit theory, amplifiers, oscillators, digital communication, digital signal processing, electrical machines.

Computer Science:

Data Structures: Stack, queue, linked list, binary tree, heap, AVL tree, B-tree.

Design and Analysis of Algorithms: Sorting, selection, searching, hashing, string handling algorithms, graph algorithms, algebraic algorithms, geometric algorithms, NP-completeness.

Programming Languages: Fundamental concepts - abstract data types, procedure call and parameter passing, C-like languages

Computer Organization and Architecture: Number representation, computer arithmetic, memory organization, I/O organization, microprogramming, pipelining, instruction level parallelism.

Operating Systems: Memory management, processor management, critical section, deadlocks, device management.

Principles of Compiler Construction: Lexical analyzer, symbol table, parser, code optimization.

Formal Languages and Automata Theory: Finite automata and regular expression, context-free grammars, Turing machines, elements of undecidability.

Database Systems: ER diagram, relational model, relational algebra, relational calculus, functional dependency, multivalued dependency, normalization (upto 4th normal form), concurrency control, crash recovery.

Computer Networks: Layered network structures, network security, LAN technology - Bus/tree, Ring, Star; ALOHA, CSMA, CSMA-CD; WAN technology - Circuit switching, packet switching; data communications - data encoding, flow control, error detection/correction.

Sample Questions

Note that all questions in the sample set are not of equal difficulty. They may not carry equal marks in the test.

GROUP A

ELEMENTS OF COMPUTING

A1. There is a counterfeit coin in a group of 16 identical-looking coins. All the coins except the counterfeit one have the same weight, while the counterfeit coin is lighter than any other coin. Show how you would find the counterfeit coin by only three comparisons using a common balance.

A2. How many isomers are there for the organic compound C_6H_{14}? In other words, how many distinct non-isomorphic unlabelled trees are there with 6 vertices of degree 4, and 14 vertices of degree 1?

A3. In the following table, find the entry in the square marked with $*$. Justify your answer.

BD_1	CE_5	DF_{21}
EG_2	FH_8	GI_{34}
HJ_3	IK_{13}	$*$

A4. Consider the pseudo-code given below.

Input: Integers b and c.
1. $a_0 \leftarrow \max(b, c)$, $a_1 \leftarrow \min(b, c)$.
2. $i \leftarrow 1$.
3. Divide a_{i-1} by a_i. Let q_i be the quotient and r_i the remainder.
4. If $r_i = 0$ then go to Step 8.
5. $a_{i+1} \leftarrow a_{i-1} - q_i * a_i$.
6. Increment i by 1.
7. Go to Step 3.
8. Print a_i.

What does the above algorithm do? What is the mathematical relation between the output a_i and the two inputs b and c?

A5. Write the output of the following pseudo-code:

```
for (n = 15, downto 2, step -2)
    if (n > 10)
        then n <- n + 1 and print n;
        else n <- n - 1 and print n;
endfor
```

A6. Given an array of n integers, write a pseudo-code for reversing the contents of the array without using another array. For example, for the array
10 15 3 30 3
the output should be
3 30 3 15 10.
You may use one temporary variable.

GROUP B

(i) MATHEMATICS

M1. (a) Let $f : I\!\!R \to I\!\!R$ be a differentiable function for which there does not exist any $x \in [0, 1]$ such that both $f(x) = 0$ and $f'(x) = 0$. Show that f has only a finite number of zeros in $[0, 1]$.
[$f'(x)$ denotes the derivative of f at x].

(b) Let $f : I\!\!R \to I\!\!R$ be such that $f'(x)$ exists and is continuous in $[0, \infty)$. Show that

$$\lim_{x \downarrow 0} \frac{1}{x^2} \int_0^x (x - 3y) f(y) dy = -\frac{f(0)}{2}.$$

[$x \downarrow 0$ denotes: x decreases to zero.]

M2. (a) Let $f : [0, 1] \to [0, 1]$ be such that

$$f(x) = nx - [nx]; \frac{1}{n} < x \leq \frac{1}{n-1}; \quad n = 2, 3, 4, \ldots; \ x \neq 0$$

and $f(0) = 0$. Show that $\int_0^1 f(x)dx$ exists and find its value. Note: $[y]$= Largest integer $\leq y; y \in I\!\!R$.

(b) Let $f : [0,1] \to (0,\infty)$ be continuous. Let

$$a_n = (\int_0^1 (f(x))^n dx)^{\frac{1}{n}}; \quad n = 1, 2, 3, \ldots$$

Find $lim_{n\to\infty} a_n$.

M3. (a) Find the Fourier transform of the unit impulse (Dirac's delta function $\delta(t)$).

(b) Find the Laplace transform of the periodic function $f(t)$ defined by

$$f(t) = (t - n\lambda)^2, \quad \text{for } n\lambda \leq t \leq (n+1)\lambda, \; n \geq 0, \; \lambda > 0.$$

M4. (a) Let u be a function of x, y and z satisfying the partial differential equation

$$(y - z)\frac{\partial u}{\partial x} + (z - x)\frac{\partial u}{\partial y} + (x - y)\frac{\partial u}{\partial z} = 0.$$

Show that u is of the form, $u = \psi(x + y + z, x^2 + y^2 + z^2)$, for some function ψ.

(b) Prove that

$$(4x + 3y + 1)dx + (3x + 2y + 1)dy = 0$$

represents hyperbolas having the following lines as asymptotes:

$$x + y = 0, 2x + y = 0.$$

M5. (a) Let $f : [a,b] \to I\!R$ be such that f is continuous on $[a,b]$, f has a finite derivative on (a,b) and $f(a) = f(b) = 0$.

Prove that for every real λ, there is some $c \in (a,b)$ such that $f'(c) = \lambda f(c)$.

(b) Let Σa_n be a convergent series such that $a_n \geq 0$ for all n. Show that $\sum \frac{\sqrt{a_n}}{n^p}$ converges for every $p > \frac{1}{2}$.

M6. (a) Prove that any finitely generated subgroup of $(Q, +)$ is cyclic, where $(Q, +)$ is the group of rational numbers with usual addition operation $+$.

(b) Prove that $Aut(Q, +) \cong Z_2$ where Z_2 is the group consisting of only two elements; and $Aut(Q, +)$ is the automorphism group of $(Q, +)$.

M7. Let $R = (S, +, ., 0, 1)$ be a commutative ring and n be a positive integer such that $n = 2^k$ for some positive integer k.

(a) Show that for every $a \in S$

$$\sum_{i=0}^{n-1} a^i = \prod_{i=0}^{k-1} (1 + a^{2^i}).$$

(b) Let $m = w^{\frac{n}{2}} + 1$ where $w \in S, w \neq 0$. Then show that for $1 \leq p < n$,

$$\sum_{i=0}^{n-1} w^{ip} \equiv 0 \bmod m.$$

M8. (a) Show that there is a basis consisting of only symmetric and skew-symmetric matrices for the vector space of all $n \times n$ matrices over $\mathbb{R}$.

(b) Does there exist a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ such that $(0, 1, 1), (1, 1, 0), (1, 2, 1)$ are mapped to $(1, 0, 0), (0, 1, 0), (0, 1, 1)$ respectively. Justify your answer.

M9. (a) Let A be a 3×3 real symmetric matrix such that the trace of A is 6 and the determinant of A is 18. Show that A cannot be a positive definite matrix.

(b) Solve the system

$$\begin{aligned} 2x_1 + x_2 + 5x_3 &= 4 \\ 3x_1 - 2x_2 + 2x_3 &= 2 \\ 5x_1 - 8x_2 - 4x_3 &= 1. \end{aligned}$$

M10. Let k be a positive integer. Let $G = (V, E)$ be the graph where V is the set of all strings of 0's and 1's of length k, and $E = \{(x, y) : x, y \in V,\ x \text{ and } y \text{ differ in exactly one place}\}$.
(i) Determine the number of edges in G.
(ii) Prove that G has no odd cycle.
(iii) Prove that G has a perfect matching.
(iv) Determine the maximum size of an independent set in G.

M11. (a) A *Tournament* T is a directed graph in which for every pair of vertices $\{u, v\}$, either $\{u, v\}$ or $\{v, u\}$ is an arc in T. Prove that every tournament has a Hamiltonian path (that is, a spanning directed path).

(b) Give an example of a graph without K_4 whose chromatic number is 4.

M12. (a) Show that, given $2^n + 1$ points with integer coordinates in R^n, there exists a pair of points among them such that all the coordinates of the midpoint of the line segment joining them are integers.

(b) Find the number of functions from the set $\{1, 2, 3, 4, 5\}$ *onto* the set $\{1, 2, 3\}$.

M13. (a) Solve the following linear programming problem:

Maximize $Z = 2x_1 + x_2$

subject to

$$\begin{aligned} 2x_1 + 5x_2 &\leq 17 \\ 3x_1 + 2x_2 &\leq 10 \\ x_1 &\geq 0. \end{aligned}$$

(b) Consider the following two linear programming problems:

P_1: Minimize x_1 subject to

$$\begin{aligned} x_1 + x_2 &\geq 1 \\ -x_1 - x_2 &\geq 1 \end{aligned}$$

where both x_1 and x_2 are unrestricted.

P_2: Minimize x_1 subject to

$$\begin{aligned} x_1 + x_2 &\geq 1 \\ -x_1 - x_2 &\geq 1 \end{aligned}$$

where $x_1 \geq 0$ and $x_2 \geq 0$.

Write the duals D_1 of P_1 and D_2 of P_2. Solve those among P_1, P_2, D_1 and D_2 which are feasible.

M14. (a) A set S contains integers 1 and 2. S also contains all integers of the form $3x + y$ where x and y are distinct elements of S, and every element of S other than 1 and 2 can be obtained as above. What is S? Justify your answer.

(b) Let $\phi(n)$ denote the number of positive integers m relatively prime to n; $m < n$.

Let $n = pq$ where p and q are prime numbers. Then show that

$$\phi(n) = (p-1)(q-1) = pq\left(1 - \frac{1}{q}\right)\left(1 - \frac{1}{p}\right).$$

(ii) STATISTICS

S1. (a) Let $\{X_n\}_{n\geq 1}$ be a sequence of random variables satisfying
$X_{n+1} = X_n + Z_n$ (addition is modulo 5),
where $\{Z_n\}_{n\geq 1}$ is a sequence of independent and identically distributed random variables with common distribution
$P(Z_n = 0) = 1/2,\ P(Z_n = -1) = P(Z_n = +1) = 1/4.$
Assume that X_1 is a constant belonging to $\{0, 1, 2, 3, 4\}$. What happens to the distribution of X_n as $n \to \infty$?

(b) Let $\{Y_n\}_{n\geq 1}$ be a sequence of independent and identically distributed random variables with a common uniform distribution on $\{1, 2, ..., m\}$. Define a sequence of random variables $\{X_n\}_{n\geq 1}$ as $X_{n+1} = MAX\{X_n, Y_n\}$ where X_1 is a constant belonging to $\{1, 2, ..., m\}$. Show that $\{X_n\}_{n\geq 1}$ is a Markov chain and classify its states.

S2. Let there be r red balls and b black balls in a box. One ball is removed at random from the box. In the next stage $(a + 1)$ balls of the color same as that of the removed ball were put into the box $(a \geq 1)$. This process was repeated n times. Let X_n denote the total number of red balls at the n-th instant.

(a) Compute $E(X_n)$.

(b) Show that if $(r + b)$ is much larger than a and n,

$$\frac{1}{r}E(X_n) = (1 + \frac{na}{r + b}) + O(\frac{1}{r + b}).$$

S3. Let $x_1, x_2, \ldots, x_n$ be a random sample of size n from the gamma distribution with density function

$$f(x, \theta) = \frac{\theta^k}{\Gamma(k)} e^{-\theta x} x^{k-1}, \quad 0 < x < \infty,$$

where $\theta > 0$ is unknown and $k > 0$ is known. Find a minimum variance unbiased estimator for $\frac{1}{\theta}$.

S4. Let $0 < p < 1$ and $b > 0$. Toss a coin once where the probability of occurrence of head is p. If head appears, then n independent and identically distributed observations are generated from Uniform$(0, b)$ distribution. If the outcome is tail, then n independent and identically distributed observations are generated from Uniform$(2b, 3b)$ distribution. Suppose you are given these n observations $X_1, ..., X_n$, but not the outcome of the toss. Find the maximum likelihood estimator of b based on $X_1, ..., X_n$. What happens to the estimator as n goes to ∞ ?

S5. Let $X_1, X_2, \ldots,$ be independent and identically distributed random variables with common density function f. Define the random variable N as

$$N = n, \text{ if } X_1 \geq X_2 \geq \ldots \geq X_{n-1} < X_n; \text{ for } n = 2, 3, 4, \ldots$$

Find $Prob(N = n)$. Find the mean and variance of N.

S6. (a) Let X and Y be two random variables such that

$$\begin{pmatrix} logX \\ logY \end{pmatrix} \sim N(\mu, \Sigma).$$

Find a formula for $\varphi(t, r) = E(X^t Y^r)$, where t and r are real numbers, and E denotes the expectation.

(b) Consider the linear model
$y_{n\times 1} = A_{n\times p}\beta_{p\times 1} + \varepsilon_{n\times 1}$ and the usual Gauss-Markov set up where, $E(\varepsilon) = 0$ and $D(\varepsilon) = \sigma^2 I_{n\times n}$, E denotes *Expectation* and D denotes *dispersion*.
Assume that A has full rank. Show that $Var(\beta_1^{LS}) = (\alpha - \Gamma^T B^{-1}\Gamma)^{-1}\sigma^2$ where

$$A^T A = \begin{bmatrix} \alpha_{1\times 1} & \Gamma^T_{1\times \overline{p-1}} \\ & \\ \Gamma_{\overline{p-1}\times 1} & B_{\overline{p-1}\times\overline{p-1}} \end{bmatrix}$$

and β_1^{LS} = the least square estimate of β_1, the first component of the vector β. Var denotes the *variance* and ^{T} denotes transpose.

S7. Let $p_1(x)$ and $p_2(x)$ denote the probability density functions for classes 1 and 2 respectively. Let P and $(1 - P)$ be the prior probabilities of the classes 1 and 2, respectively. Consider

$$p_1(x) = \begin{cases} x, & x \in [0, 1); \\ 2 - x, & x \in [1, 2]; \\ 0, & \text{otherwise}; \end{cases}$$

and

$$p_2(x) = \begin{cases} x - 1, & x \in [1, 2); \\ 3 - x, & x \in [2, 3]; \\ 0, & \text{otherwise}. \end{cases}$$

(i) Find the optimal Bayes risk for this classification problem.

(ii) For which values of P, is the above risk

(I) minimized ?

(II) maximized ?

S8. Let $\mathbf{X} = (X_1, ...X_n)$ and $\mathbf{Y} = (Y_1, ..., Y_n)$ be two independent and identically distributed multivariate random vectors with mean $\mathbf{0}$ and covariance matrix $\sigma^2 \mathbf{I_n}$, where $\sigma^2 > 0$ and $\mathbf{I_n}$ is the $n \times n$ identity matrix.

(a) Show that $\frac{\mathbf{X^T Y}}{\|\mathbf{X}\| \, \|\mathbf{Y}\|}$ and $V = \sum (X_i^2 + Y_i^2)$ are independent. (Here, $\|(a_1, ..., a_n)\| = \sqrt{a_1^2 + ... + a_n^2}$).

(b) Obtain the probability density of $\dfrac{\sum_{i=1}^{n} X_i^2}{\sum_{i=1}^{n} Y_i^2}$.

S9. Let $X_1, X_2, \ldots, X_n$ be independent random variables. Let $E(X_j) = j\theta$ and $V(X_j) = j^3\sigma^2$, $j = 1, 2, \ldots, n, -\infty < \theta < \infty$ and $\sigma^2 > 0$. Here $E(X)$ denotes expectation and $V(X)$ denotes the variance of the random variable X. It is assumed that θ and σ^2 are unknown.

(i)Find the best linear unbiased estimator for θ.

(ii) Find the uniformly minimum variance unbiased estimate for θ under the assumption that X_i's are normally distributed; $1 \leq i \leq n$.

S10. A hardware store wishes to order Christmas tree lights for sale during Christmas season. On the basis of past experience, they feel that the demand v for lights can be approximately described by the probability density function $f(v)$. On each light ordered and sold they make a profit of a cents, and on each light ordered but not sold they sustain a loss of b cents. Show that the number of lights they should order to maximize the expected profit is given by x, which is the solution of the equation:

$$\int_0^x f(v)dv = \frac{a}{a+b}$$

S11. Let (X, Y) follow the bivariate normal distribution. Let *mean* of X = *mean* of $Y = 0$. Let *variance* of X = *variance* of $Y = 1$, and the *correlation coefficient* between X and Y be ρ. Find the correlation coefficient between X^3 and Y^3.

S12. Let X have probability density function $f(x)$ $(-\infty < x < \infty)$, and we have two hypotheses $H_O : f(x) = (2\pi)^{-1/2} \exp(-x^2/2)$ against $H_A : f(x) = (1/2) \exp(-|x|)$. Derive the most powerful test at level $\alpha = 0.05$.

S13. Let $X_1, X_2, \ldots .X_n$ be independent and identically distributed observations with a common exponential distribution with mean μ. Show that there is no uniformly most powerful test for testing $H_O : \mu = 1$ against $H_A : \mu \neq 1$ at a given level $0 < \alpha < 1$ but there exists a uniformly most powerful unbiased test and derive that test.

S14. (a) An unbiased die is rolled once. Let the score be $N \in \{1, 2, \ldots, 6\}$. The die is then rolled N times. Let X be the maximum of these N scores. Find the probability of the event $(X = 6)$.

(b) The unit interval $(0, 1)$ is divided into two sub-intervals by picking a point at random from the interval. Denote by Y and Z the lengths of the longer and shorter sub-intervals, respectively. Find the distribution of Z and show that $\frac{Y}{Z}$ does not have a finite expectation.

S15. Let $X_1,\ X_2,\ X_3$ be independent and identically distributed observations with a common double exponential distribution with density

$$f(x, \theta) = \frac{1}{2} \exp(- \mid x - \theta \mid), -\infty < x < \infty, -\infty < \theta < \infty.$$

Suppose that the observations are all distinct.

(a) Find a maximum likelihood estimator of θ. Give a complete argument for your answer.

(b) Suppose it is known that $-10 \leq \theta \leq 10$. Find a maximum likelihood estimator of θ. Justify your answer.

S16. Consider the following linear model

$$y_{ij} = \alpha_i + \beta_j + e_{ij},\ \ i = 1, 2;\ \ j = 1, 2, 3;$$

(a) What is the rank of the error-space? Justify your answer.

(b) Write down any linear function of observations that belongs to (i) estimation space, (ii) error space.

(c) Write down a parametric function that is not estimable. Justify your answer.

S17. Let $A = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$ be the space obtained by tossing a coin three times.
Let $f : A \to (0, \infty)$ and $x_1 \in A$. For any $x_i \in A$, x_{i+1} is found in the following way.
Toss a fair coin three times and let the outcome be z.

If $f(z) \geq f(x_i)$ then $x_{i+1} = z$, otherwise $x_{i+1} = x_i$.

What can you say about $\lim_{i \to \infty} f(x_i)$? Justify your answer.

S18. Let there be two classes C_1 and C_2. Let the density function for class C_i be p_i for $i = 1, 2$ where $p_i(x) = ie^{-ix}$; $x > 0$, $i = 1, 2$. Let the prior probability for C_1 be 0.4 and the prior probability for C_2 be 0.6. Find the decision rule for classification of an observation, which provides minimum probability of misclassification and find its value for that decision rule.

(iii) PHYSICS

P1. Consider an ideal gas confined in an insulated chamber of volume V_1 at a pressure P_1 and temperature T_1. Let this gas expand into an insulated evacuated vessel of volume V_2 until it fills both vessels. Show that the volume V_2 be expressed as

$$V_2 = V_1(e^{\Delta\Phi/R} - 1)$$

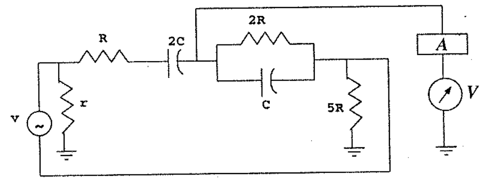
where $\Delta\Phi$ is the change in entropy.

P2. In studying the emission of electrons from metals it is necessary to take into account the fact that electrons with energy sufficient to escape from the metal can, according to quantum mechanics, undergo reflection at the surface of the metal. Consider a one dimensional model with the potential $V = -V_0$ for $x < 0$ (inside the metal), and $V = 0$ for $x > 0$ (outside the metal), and determine the reflection coefficient of an electron of energy $E > 0$ at the surface of the metal.

P3. Let $\vec{E}$ and $\vec{B}$ be the electric field and the magnetic induction field, respectively at a certain point in space and time in a system K, and let $\vec{E}'$ and $\vec{B}'$ be the corresponding fields at the same point in space but in another system K', moving relative to the system K at a velocity v directed along the X-axis. Write down the expressions for $\vec{E}'$ and $\vec{B}'$ in terms of $\vec{E}$ and $\vec{B}$. Show also that $\vec{E}.\vec{B}$ and $E^2 - c^2B^2$ remain invariant under the Lorentz transformation.

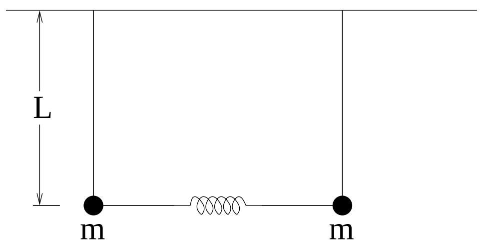
P4. Calculate the resultant of two rectangular simple harmonic vibrations whose amplitudes as well as periods are in the ratio 2:1, and the phase difference is 90^0.

P5. Consider the circuit shown in figure below. The input supply v is a variable frequency voltage source. A is a high precision ac amplifier whose output is connected to a voltmeter V.



Find the value of r and the angular frequency ω of the input, at which the voltmeter reading would be zero. Assume that $R = 1000\Omega$ and $c = 0.01\mu F$.

P6. Two simple pendulums, each of length L and mass m, are connected by a weightless spring as shown in the figure below. In the equilibrium position the spring is not deformed. Find the frequency of small oscillations when they are deflected in the same plane through the same angle in opposite directions.



P7. Calculate the numerical relationship between the atmospheric thickness (difference in height in m) from 1000 mb surface to 500 mb surface and the mean virtual temperature of the layer from 1000 to 500 mb (in ^{0} C). If thickness lines are drawn at the intervals of 100 geopotential meter, to what mean temperature interval does this correspond?

P8. In the physical interpretation of quantum mechanics, the eigenvalues of the energy are the only values obtainable by an experimental measurement of the energy. The operator a^+a, which is hermitian, is called the number operator where a and a^+ are the annihilation and creation operators respectively with the commutation relation

$$[a, a^+] = 1$$

Solve

$$a^+a|n >= n|n >$$

to show that $\{|n>\}$ forms a complete orthonormal set of basis vectors for a Hilbert space.
Show that

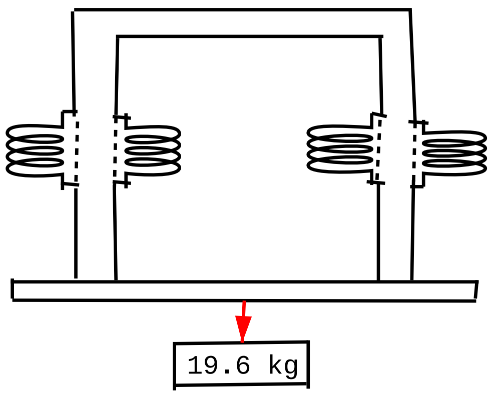
$$\begin{aligned} a|n> &= \sqrt{n}|n-1> \\ a^{+}|n> &= \sqrt{n+1}|n+1> \end{aligned}$$

Hence find the solution of

$$a|\alpha>=\alpha|\alpha>$$

where $|\alpha>=\sum_{n=0}^{\infty} c_n|n>$

P9. A horse-shoe magnet is formed out of a bar of wrought iron of 50 cm length having cross section 6.28 cm^2. Exciting coils of 500 turns are placed on each limb and connected in series. Find the exciting current necessary for the magnet to lift a load of 19.6 kg (see the figure given below) assuming that the load has negligible reluctance and makes close contact with the magnet. Relative permeability of iron is 700.



P10. Consider a particle of mass m and energy E approaching a potential barrier V where

$$V = \begin{cases} 0, & x < 0; \\ V_0, & 0 \le x \le d; \\ 0, & x > d. \end{cases}$$

Show that the transmission co-efficient, T, is given approximately by
$T \simeq exp(-2d\sqrt{\frac{2m(V-E)}{(\frac{h}{2\pi})^2}})$. (Assume $d\sqrt{\frac{2m(V-E)}{(\frac{h}{2\pi})^2}} >> 1$.)

P11. A solid sphere of weight W rolls without sliding down a plane inclined at an angel of θ to the horizontal. Write down the equations of motion and show that the acceleration of the center of gravity of the body is given by

$$a = \frac{g\ sin\theta}{1 + \frac{k^2}{r^2}}$$

where g is the acceleration due to gravity, k is the radius of gyration and r is the radius of the sphere.

P12. A sample of protons (in some crystals) is in a uniform external magnetic induction field $\mathbf{B}$. When the sample is irradiated by electro-magnetic waves of suitable polarisation, the maximum rate of power absorption (due to flipping of proton spins) occurs at a frequency of 100 MHz. What is the fractional polarisation P of this sample of proton spins with this field $\mathbf{B}$? Assume room temperature and

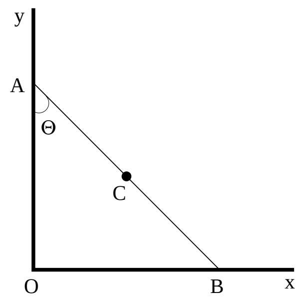
$$P = \frac{N(up) - N(down)}{N(up) + N(down)}.$$

$N(up)$ = Number of protons with spin up,
$N(down)$ = Number of protons with spin down.

P13. (a) A particle of mass m is attached to the midpoint C of a weightless rod AB of length l as shown in the figure below. Assume that the particle cannot slide. The ends of the rod are constrained to move along the x and y axes. A uniform gravitational field acts in the negative y-direction. Use Θ ($\angle OAB$ in figure) as a generalized coordinate and neglect friction.

(i) Write the Lagrangian and obtain the equation of motion.

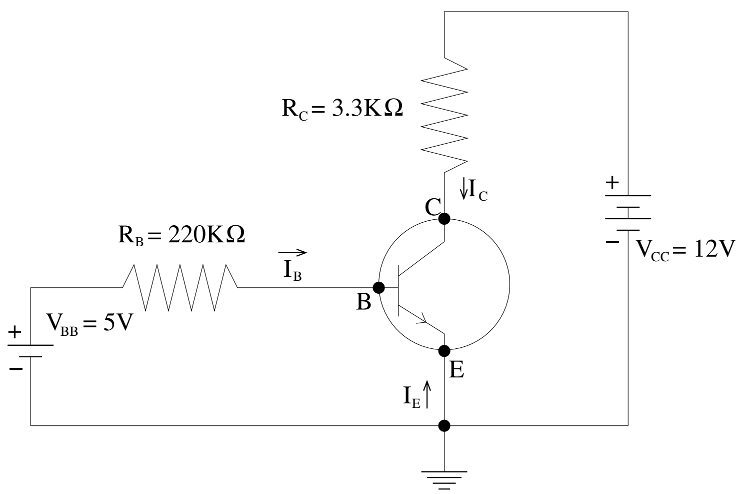
(ii) Solve the equation of motion for small Θ, *i.e.,* $\mid \Theta \mid << 1$, assuming that at $t = 0$, $\Theta = \Theta_0$ and $\frac{d\Theta}{dt} = 0$.



(b) A submarine is travelling through a vast ocean of liquid with refractive index $n = 1.36$ at a speed of $0.7c_n$ where c_n is the velocity of light in the liquid. The submarine launches a torpedo at a velocity of $0.4c_n$ in the direction in which the submarine is moving.

What is the velocity of the torpedo relative to an observer stationary at the bottom of the ocean?
(The velocity of light c in vacuum is 3×10^8 m/s.)

P14. (a) A silicon n-p-n transistor shown below having $\beta = 100$ and reverse saturation current $I_{CO} = 22$nA is operated in the CE configuration. Assuming $V_{BE} = 0.7$V, determine the transistor currents I_C, I_B, I_E and the region of operation of the transistor. What happens if the resistance R_C is indefinitely increased?



(b) A constant voltage is applied to n groups of resistors in series where each group has m identical resistors in parallel. One resistor burns out in one group. Find the percentage change of current in each resistor of

(i) the faulty group and
(ii) any fault-free group.

P15. Two electrons are confined in a one dimensional box of length a. A clever experimentalist has made arrangements such that both the electrons are in the same spin state. Ignore the Coulomb interaction between the electrons.

(a) Write down the ground state wave function of the two-electron system.

(b) What is the probability that both the electrons are found on the same half of the box?

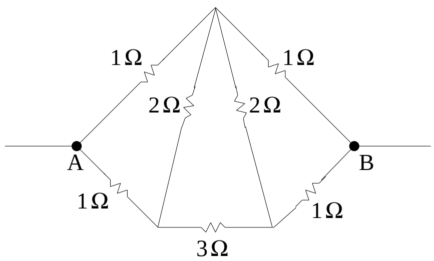
(c) Will the nature of construction of the wave function in (a) hold if Coulomb interaction is included? Give reasons for your answer.

(d) In the above problem, consider two charged π-mesons instead of two electrons. Write down the ground state wave function ignoring the Coulomb interactions.

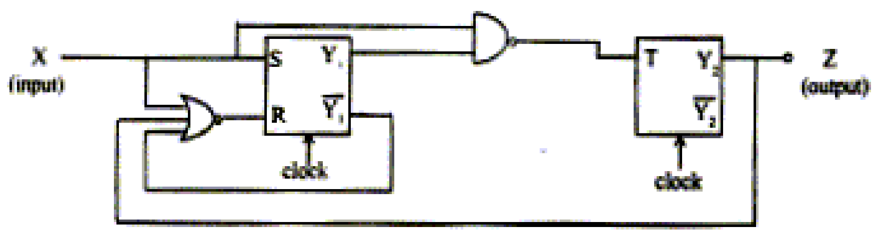
(iv) RADIOPHYSICS/TELECOMMUNICATIONS/ELECTRONICS/ ELECTRICAL ENGINEERING

R1. Design a sequential machine that produces an output 1 whenever a substring of 5 consecutive symbols in the input starts with two 1's and contains exactly three 1's. If a substring of 5 symbols starts with two 1's, the analysis of the next substring does not begin until the processing of the current substring is complete. Realize this circuit with minimum number of NAND gates and flip flops.

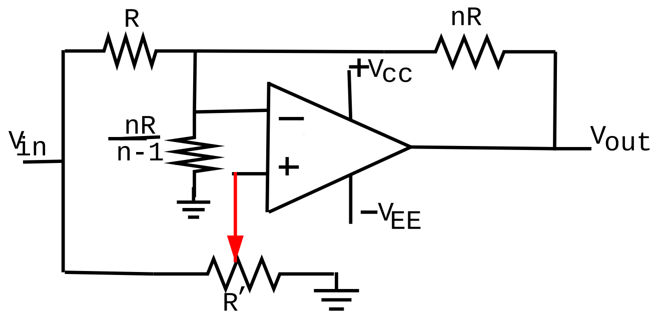
R2. A network of resistances is shown in the following figure. Find the equivalent resistance between the points A and B.



R3. Draw the state table for the synchronous sequential circuit shown in the figure below:



R4. Calculate the range of voltage gain of the following circuit when the variable resistance R' changes from minimum to maximum.



R5. Consider a voltage amplifier circuit shown in figure below, where R_i and R_0 represent the input and output impedances respectively, C_0 is the total parasitic capacitance across the output port, μ is the amplifier gain and the output is terminated by a load resistance R_L.

(i) Calculate the current, voltage and power gain in decibels (dB) of the amplifier, when

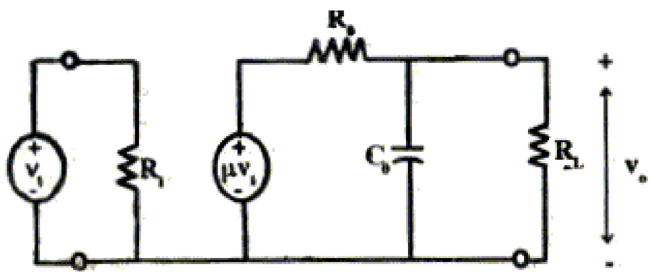
$R_i = 1M\Omega$; $R_L = 600\Omega$; $R_o = 100M\Omega$,

$C_o = 10pf$; $\mu = 10$.

(ii) Calculate 3-dB cutoff frequency of the amplifier when

$R_i = 5K\Omega$; $R_L = 1K\Omega$; $R_o = 100\Omega$

$C_o = 10pf$; $\mu = 2$.



R6. (a) Find the Fourier transform of the point spread function

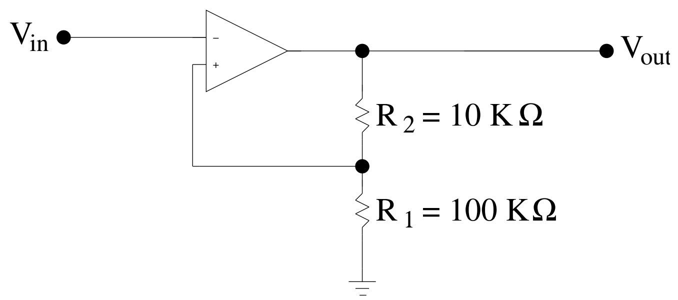
$$h(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

and show that it is rotationally symmetric.

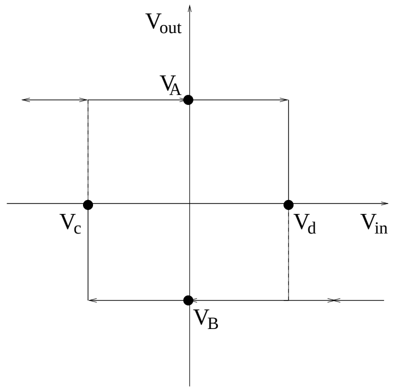
(b) Show that if a signal is passed through the above filter function, high frequencies will be more attenuated in amplitude compared to low frequencies.

R7. A 440 *volt* DC shunt motor has an armature resistance of 0.5 Ω and a field resistance of 220 Ω. The motor takes 6 *amp* of current when idle on a 440 *volt* line. Calculate the efficiency of the motor at full load, when the current is 75 *amp*.

R8. Consider the following circuit with an OP-AMP.



The plot of output voltage V_{out} vs. input voltage V_{in} for the given circuit is as follows.



Let $V_A = 10\ V$ and $V_B = -10\ V$. Assume that $V_{in} < V_c$, and is gradually increasing. The output voltage $V_{out} = V_A$ until $V_{in} = V_d$ and then falls to V_B. The output remains at V_B for $V_{in} > V_d$. Similarly, if V_{in} is initially $> V_d$ and gradually reduced, V_{out} remains at V_B until $V_{in} = V_c$, and then rises to V_A for all values $V_{in} < V_c$.

(i) Explain why the circuit behaves in this fashion, and

(ii) calculate the values of V_c and V_d.

R9. Assume that an analog voice signal which occupies a band from 300 Hz to 3400 Hz, is to be transmitted over a Pulse Code Modulation (PCM) system. The signal is sampled at a rate of 8000

samples/sec. Each sample value is represented by 7 information bits plus 1 parity bit. Finally, the digital signal is passed through a raised cosine roll-off filter with the roll-off factor of 0.25. Determine
(i) whether the analog signal can be exactly recovered from the digital signal;
(ii) the bit duration and the bit rate of the PCM signal before filtering;
(iii) the bandwidth of the digital signal before and after filtering;
(iv) the signal to noise ratio at the receiver end (assume that the probability of bit error in the recovered PCM signal is zero).

R10. A causal LTI discrete-time system develops an output

$$y[n] = (0.4)^n u[n] - (0.3)(0.4)^{n-1} u[n-1]$$

for an input $x[n] = (0.2)^n u[n]$.
(i)Determine the transfer function of the system and also the difference equation characterizing the system.
(ii) Develop a canonical direct form II realization of the system with no more than three multipliers OR a Parallel Form I realization of the system.
(iii) Determine the impulse response of the system.
(iv) Determine the output $y[n]$ of the system for an input $x[n] = (0.3)^n u[n] - (0.4)(0.3)^{n-1} u[n-1]$.

R11. You are presented with a set of requirements under which an insurance policy will be issued. The applicant must be:
a) A married female 25 years old or over, or
b) A female under 25, or
c) A married male under 25 who has not been involved in a car accident, or
d) A married male who has been involved in a car accident, or
e) A married male 25 years or over who has not been involved in a car accident.

(i) Find an algebraic expression which assumes the value 1 whenever the policy should be issued.
(ii) Simplify the above expression algebraically.
(iii) Realize the function using minimum number of NAND gates.

R12. A message $bbccfe\backslash$ needs to be encoded using arithmetic coding. The probabilities of message symbols are shown in the following table.

symbol	a	b	c	d	e	f	\
Probability	0.05	0.2	0.1	0.05	0.3	0.2	0.1

Using the symbol probabilities shown in the above table, find

(a) a fractional value that is to be transmitted after encoding the message $bbccfe\backslash$,

(b) the exact decoding of the message from the fractional value estimated at the encoding stage, and

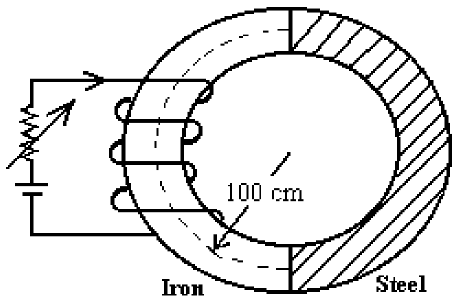
(c) the number of bits required to represent the encoded message after arithmetic coding.

R13. (a) A long shunt d.c. generator supplies a load of two motors each drawing 46 ampere and a load consisting of twenty-two 60 Watt lamps at 220V. The resistances of shunt field, series field and armature are 110Ω, 0.06Ω and 0.05Ω, respectively.

(i) Find the electrical efficiency of the generator.

(ii) If the overall efficiency of the generator at the above load is 77%, find the iron and mechanical loss together.

(b) Suppose a metal ring of mean radius 100 cm is made of iron and steel as shown in the following figure.



Cross-section of the ring is 10 cm^2. If the ring is uniformly wound with 1000 turns, calculate the current required to produce a flux of 1 milliweber. Absolute permeability of air is $4\pi \times 10^{-7}$H/m and relative permeabilities of iron and steel are 250 and 1000, respectively.

R14. (a) Consider a signal

$$x[n] = \begin{cases} 1, & 0 \leq n \leq 5 \\ 0, & \text{otherwise.} \end{cases}$$

Let $X(z)$ denote the Z-transform and $X[k]$ denote the Discrete Fourier Transform (DFT) of $x[n]$. If

$$X[k] = X(z)\mid_{z=e^{j(2\pi/4)k}}, k = 0, 1, 2, 3,$$

compute the inverse DFT (IDFT) of $X[k]$ without explicitly using the formulae for DFT or IDFT.

(b) Determine the impulse response of the stable and causal system whose difference equation is

$$y[n] = \frac{3}{4}y[n-1] - \frac{1}{8}y[n-2] + x[n] - x[n-1].$$

(c) For a signal

$$x[n] = \begin{cases} 1, & n = 0, 2 \\ -1, & n = 1, 3 \\ 0, & \text{otherwise} \end{cases}$$

determine the DFT. Using this DFT, determine the DFT of

$$y[n] = \begin{cases} -1, & n = 0, 2 \\ 1, & n = 1, 3 \\ 0, & \text{otherwise.} \end{cases}$$

(v) COMPUTER SCIENCE

C1. Consider a collection of n binary strings $S_1, \ldots, S_n$. Each S_i is of length l_i bits where $1 \le l_i \le k$.

(a) Write a function `prefix(S,T)` in C programming language that takes two binary strings S, T and returns 1 if S is a prefix of T, else it returns 0. For example `prefix(001,00101)` returns 1 but `prefix(010,00101)` returns 0.

(b) Suppose we want to report all the pairs (i, j) for which S_i is a prefix of S_j $(1 \le i \ne j \le n)$. How many times do we need to call the `prefix` function described above?

(c) Present an $O(nk)$ time algorithm to report all the (i, j)'s as mentioned in (b). (Hint: Use a binary tree with each edge marked as 0 or 1; a path from the root to a node in the tree represents a binary string.)

C2. Let $S = \{x_1, x_2, ..., x_n\}$ be a set of n integers. A pair (x_i, x_j) is said to be the closest pair if $|x_i - x_j| \le |x_{i'} - x_{j'}|$, for all possible pairs $(x_{i'}, x_{j'})$, $i', j' = 1, 2, ..., n, i' \ne j'$.

(a) Describe a method for finding the closest pair among the set of integers in S using $O(n\ log_2 n)$ comparisons.

(b) Now suggest an appropriate data structure for storing the elements in S such that if a new element is inserted to the set S or an already existing element is deleted from the set S, the current closest pair can be reported in $O(log_2 n)$ time.

(c) Briefly explain the method of computing the current closest pair, and necessary modification of the data structure after each update. Justify the time complexity.

C3. Let A be an $n \times n$ matrix such that for every 2×2 sub-matrix $\left[\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right]$ of A, if $a < b$ then $c \leq d$.

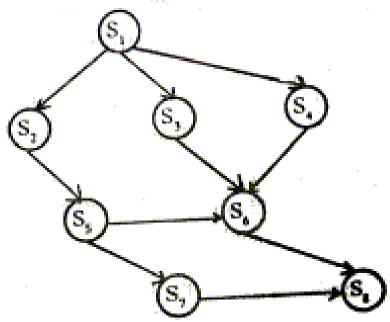
Note that for every pair of rows i and j, if a_{ik} and $a_{j\ell}$ are the largest elements in i-th and j-th rows of A, respectively, then $k \leq \ell$ (as illustrated in the 5×5 matrix below).

$$\begin{bmatrix} 3 & 4 & 2 & 1 & 1 \\ 7 & 8 & 5 & 6 & 4 \\ 2 & 3 & 6 & 6 & 5 \\ 5 & 6 & 9 & 10 & 7 \\ 4 & 5 & 5 & 6 & 8 \end{bmatrix}$$

(i) Write an algorithm for finding the maximum element in each row of the matrix with time complexity $O(n\log n)$.

(ii) Establish its correctness, and justify the time complexity of the proposed algorithm.

C4. (a) Consider the precedence graph shown in figure below.



Can this precedence graph be expressed using only concurrent statements? If so, how? If not, why? How can this precedence graph be expressed if semaphores are also used?

C5. Consider a file consisting of 100 blocks. Assume that each disk I/O operation accesses a complete block of the disk at a time. How many disk I/O operations are involved with contiguous and linked allocation strategies, if one block is (i) added at the beginning? (ii) added at the middle? (iii) removed from the beginning? (iv) removed from the middle?

C6. (a) Let M_1 be a deterministic finite automation which accepts a language $L_1 \subseteq \{0,1\}^*$. Define

$$L_2 = \{xb : b \in \{0,1\}, x \in L_1 \text{ and } xb \in L_1\}.$$

Construct a deterministic finite automation M_2 which accepts L_2. Show that if M_1 has n states, it is sufficient for M_2 to have $2n$ states.

(b) Consider the grammar $G = (V,\ T,\ P,\ S)$, where $V = \{S,\ A_1,\ A_2,\ B_1,\ B_2\}$, $T = \{a,\ b\}$ and P consists of the productions given below. (Note that ϵ denotes the empty string.)

$$\begin{array}{lcl} S & \rightarrow & A_1B_1|A_2B_2 \\ A_1 & \rightarrow & aA_1|\epsilon \\ A_2 & \rightarrow & aaA_2|\epsilon \\ B_1 & \rightarrow & bbB_1|b \\ B_2 & \rightarrow & bbB_2|\epsilon \end{array}$$

Show that for each $i, j \geq 0$, the string a^ib^j has at most one left-most derivation in G.

C7. (a) Five batch jobs $P_1, \ldots, P_5$ arrive almost at the same time. They have estimated run times of $10, 6, 2, 4,$ and 8 ms. Their priorities are $3, 5, 2, 1,$ and 4 respectively, where 1 indicates the highest priority and 5 indicates the lowest. Determine the average turn-around and waiting time for the following scheduling algorithms:

(i) Round robin with time quantum of 5 ms,

(ii) Priority scheduling.

(b) The access time of a cache memory is 100 ns and that of main memory is 1000 ns. It is estimated that 80% of the memory requests are for read and the remaining 20% are for write. The hit ratio for read access is 0.9. A write through procedure is used.

(i) What is the average access time of the system considering only memory read cycles?

(ii) What is the average access time of the system considering both read and write requests?

C8. (a) A program P consisting of 1000 instructions is run on a machine at $1GHz$ clock frequency. The fraction of floating point (FP) instructions is 25%. The average number of clock-cycles per instruction (CPI) for FP operations is 4.0, and that for all other instructions is 1.0.

(i) Calculate the average CPI for the overall program P.

(ii) Compute the execution time needed by P in seconds.

(b) Consider a $100mbps$ token ring network with 10 stations having a ring latency of $50\mu s$ (the time taken by a token to make one complete rotation around the network when none of the stations is active). A station is allowed to transmit data when it receives the token, and it releases the token immediately after transmission. The maximum allowed holding time for a token (THT) is $200\mu s$.

(i) Express the maximum efficiency of this network when only a single station is active in the network.

(ii) Find an upper bound on the token rotation time when all stations are active.

(iii) Calculate the maximum throughput rate that one host can achieve in the network.

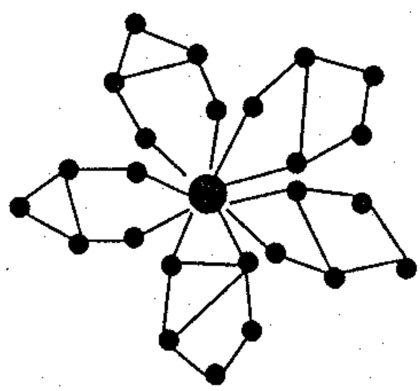
C9. An undirected graph $G = (V, E)$ with $kn + 1$ nodes is a k-daisy if it has a collection of k petals $p_1, p_2, \ldots, p_k$ $(p_i \subseteq V)$ such that

(i) $|p_i| = n + 1$

(ii) $\exists\ c \in V$ such that $p_i \cap p_j = \{c\}$ if $i \neq j$

(iii) $\forall i, \exists$ a simple cycle in G through all the vertices of p_i.

For example, see the following figure.



Prove that the decision problem of testing whether a given graph G is a 5-daisy, is NP-complete.

C10. Consider a graph G (called an interval graph) whose nodes correspond to a set of intervals on the real line. The i-th interval is denoted by $[\alpha_i, \beta_i]$, where $0 \leq \alpha_i < \beta_i$. An edge between two nodes (i, j) implies that the corresponding intervals $[\alpha_i, \beta_i]$ and $[\alpha_j, \beta_j]$ overlap.

(a) Consider the set of intervals $[3, 7]$, $[2, 4]$, $[2, 3]$, $[1, 5]$, $[1, 2]$, $[6, 7]$, $[10, 16]$, $[11, 12]$. Draw the corresponding interval graph and identify the largest subgraph where all the nodes are connected to each other.

(b) Write an algorithm which takes input the interval graph G and finds the largest subgraph of G in which all the nodes are connected to each other. What is the time complexity of your algorithm?

(c) Given a list of intervals, write an algorithm to list all the connected components in the corresponding interval graph. What is the time complexity of your algorithm?

C11. Consider the following database:

B	O	I	S	Q	D
b_1	o_1	i_1	s_1	q_1	d_1
b_2	o_2	i_2	s_2	q_2	d_2
b_1	o_1	i_3	s_3	q_3	d_3
b_3	o_3	i_4	s_2	q_1	d_2
b_4	o_1	i_5	s_4	q_4	d_4
b_2	o_2	i_2	s_3	q_2	d_3

(a) Write all the functional dependencies that hold for the above database.

(b) Derive a key for the database.

(c) If the database $R = BOSQID$ is decomposed into two schemas $ISQD$ and IBO, what redundancies and anomalies do you face?

C12. (a) A functional dependency $\alpha \to \beta$ is called a *partial* dependency if there is a proper subset γ of α such that $\gamma \to \beta$. Show that every partial dependency is a transitive dependency.

(b) Let $R = (A, B, C, D, E)$ be a schema with the set $F = \{A \to BC, CD \to E, B \to D, E \to A\}$ of functional dependencies. Suppose R is decomposed into two schema $R_1 = (A, B, C)$ and $R_2 = (A, D, E)$

(i) Is this decomposition loss-less? Justify.

(ii) Is this decomposition dependency preserving? Justify.

(c) Consider the relations $r_1(A, B, C)$, $r_2(C, D, E)$ and $r_3(E, F)$. Assume that the set of all the attributes constitute the primary keys of these relations, rather than the individual ones. Let $V(C, r_1)$ be 500, $V(C, r_2)$ be 1000, $V(E, r_2)$ be 50, and $V(E, r_3)$ be 150, where $V(X, r)$ denotes the number of distinct values that appear in relation r for attribute X. If r_1 has 1000 tuples, r_2 has 1500 tuples, and r_3 has 750 tuples, then give the ordering of the natural join $r_1 \bowtie r_2 \bowtie r_3$ for its efficient computation. Justify your answer.

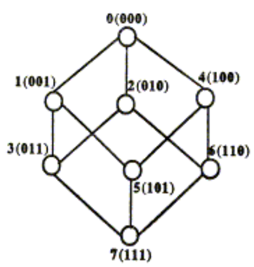
C13. (a) (i) Write a Context Free Grammar (CFG) for structure definitions in C. Assume that the only allowable types are `char`, `int`, and `float` (you need not handle pointers, arrays, structure fields, etc.).

(ii) Assume that `char`s are stored using 1 byte each; `int`s and `float`s are stored using 4 bytes each and are aligned at 4 byte boundaries. Add semantic rules to your grammar to calculate the number of bytes required to store the structure defined by your grammar.

(b) (i) Compute the canonical collection of sets of $LR(1)$ items (i.e. canonical LR items) for the following grammar:
$S \rightarrow aXcd, \quad S \rightarrow aYce, \quad X \rightarrow b, \quad Y \rightarrow b.$
Is the grammar $LR(1)$? Briefly justify.

(ii) Give an example of a grammar that is unambiguous but not $LR(2)$. Briefly justify/explain your example.

C14. Let $G_n(V, E)$ be an undirected graph, such that $|V| = 2^n$, when n is a positive integer ≥ 2; the nodes are labelled as $0, 1, 2, \ldots, 2^{n-1}$; and two nodes v_i and v_j are adjacent, i.e., $(v_i, v_j) \in E$, if their corresponding binary representations differ exactly in one bit position. For example, G_3 is shown in figure below.



Prove that
(i) $G_n(V, E)$ is a bipartite graph;
(ii) $G_n(V, E)$ admits a Hamiltonian cycle.

C15. (a) What are the conditions which must be satisfied by a solution to the *critical section* problem?

(b) Consider the following solution to the critical section problem for two processes. The two processes, P_0 and P_1, share the following variables:

```
var flag : array [0..1] of Boolean;
                    (* initially false *)
    turn : 0..1;
```

The program below is for process P_i ($i = 0$ or 1) with process P_j ($j = 1$ or 0) being the other one.

```
repeat
  flag[i] <- true;
  while (flag[j])
    do if (turn = j)
          then begin
                 flag[i] <- false;
                 while (turn = j) do skip;
               end;
    ...
    CRITICAL SECTION
    ...
  turn <- j;
  flag[i] <- false;
   ...
   REMAINDER SECTION
   ...
until false;
```

Does this solution satisfy the required conditions?

(c) If a binary signal is sent over a 3 kHz channel whose signal-to-noise ratio is 20 dB, what is the maximum achievable data-rate?

C16. (a) Construct an AVL tree of height 5 with minimum number of nodes.

(b) Consider a B-tree of order 3.

(i) Trace the insertion of the keys a, g, f, b, k, d, h, m into an initially empty tree, in lexicographic order.

(ii) Sketch the B-tree upon deletion of keys h, d.